Research Guru
Online Journal of Multidisciplinary Subjects (ISSN : 2349-266X)
www.researchguru.net
Volume-14, Issue-4, March-2021

Bianchi Type-III String Cosmological Model with Bulk Viscosity and Time-Decaying Λ Term

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Abstract: Bianchi type-III cosmological model for a cloud of string with bulk viscosity and a variable cosmological constant Λ is investigated. To obtain the determinate model of universe, we assume that the coefficient of bulk viscosity ξ is inversely proportional to the expansion θ in the model and expansion θ in the model is proportional to the eigen value σ_2^2 of the shear tensor σ_i^j. This leads to $B = lC^n$, where l and n are constants. We also assume that the cosmological term Λ is inversely proportional to S^3, where S is the scale factor. Behaviour of the model in the presence and absence of bulk viscosity is discussed. The physical implications of the model is also discussed in detail.

Keywords: Bianchi type-III, Massive string, Viscous fluid, Cosmological constant.

1. Introduction

The exact physical situations of very early stages of formatiom of universe can be explained satisfactorily by string theory and it is still an interesting and challenging problem of cosmology.The string theory is a useful concept before the creation of the particle in the universe.The strings are nothing but the important topological stable defects due to the phase transition that occurs as the temperature lowers below some critical temperature $\left(T_{GUT} = 10^{28} K\right)$ at the very early stages $\left(t \sim 10^{-36} s\right)$ of the universe as predicted by grand unified theories (GUT) (Zel'dovich et al. [1], Kibble [2,3], Everett [4], Vilenkin [5,6]). It is believed that cosmic strings give rise to density perturbations which led to the formation of galaxies [7]. Massive closed loops of strings serve as seeds for the formation of large structures like galaxies and cluster of galaxies. While matter is accreted onto loops, they oscillate violently and lose their energy by gravitational radiation and therefore they shrink and disappear.These cosmic strings have stress-energy and coupled to the gravitational field. Therefore, the study of gravitational effects of such strings will be interesting. The general relativistic treatment of strings was initiated by Letelier [8,9] who considered the massive strings to be formed by geometric strings with particles attached along its extension. Stachel [10] has also studied massive string. Exact solutions of string cosmology for Bianchi type-II, VI0, VII and IX space times have been studied by Krori et al. [11] and Wang [12]. Bali et al. [13-15] have obtained Bianchi type-IX, type-V and type-I string cosmological models in general relativity. Tikelar and Patel [16] and Chakraborty & Chakraborty [17] have presented the exact solutions of Bianchi type-III and spherically symmetric cosmology respectively for a cloud string.

In most of the cosmological models matter distribution in the universe is satisfactorily described by a perfect fluid due to large-scale distribution of galaxies in our universe. However, a realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. Cosmological models of a fluid with viscosity play a significant role in the study of evolution of the universe. It is well known that at an early stage of universe when neutrino decoupling occurred, the matter behaved like a viscous fluid [18]. Weinberg [19] derived general formulae for bulk and shear viscosity and used these to evaluate the rate of cosmological entropy production. He deduced that the most general form of the energy-momentum tensor, allowed by rotational and space-inversion invariance, contains a bulk viscosity term proportional to the volume expansion of the model. Padmanabham et al. [20] also noted that viscosity may be relevant for the future evolution of the universe. Bali and Dave [21] have presented Bianchi type-III string cosmological model with bulk viscosity, where the constant coefficient of bulk viscosity is considered. However, it is known that the coefficient of bulk viscosity is not constant but decreases as the universe expands. Cosmological models with viscous fluid in the early universe have been widely discussed in the literature (see the works by Pradhan et al [22], Pradhan & Kumar [23], Wang [24], Bali and Deo [25], Pradhan & Lata [26], Pradhan et al. [27], Yadav et al. [28]).

In modern cosmological theories a dynamical cosmological term $\Lambda(t)$ remains a focal point of interest as it solves the cosmological constant problem in a natural way. There is significant observational evidence for the detection of Einstein's cosmological constant Λ or a component of material content of the universe that varies slowly with time to act like Λ. In the context of quantum field theory, a cosmological term Λ corresponds to the energy density of vacuum. A constant Λ can not explain the huge difference between the cosmological constant inferred from observation and the vacuum energy density resulting from quantum field theories. In an attempt to solve this problem, variable Λ was introduced such that Λ was large in the early universe and then decayed with evolution [29]. A number of models with different decay laws for the variation of cosmological term were investigated during last two decades [30-34]. In recent past, several cosmological models with time-dependent cosmological constant have been extensively discussed in the literature (for example see Singh & Kumar [35], Pradhan [36], Pradhan et al. [37], Pradhan & Pandey [38], Chawla et al. [39], Tiwari et al. [40], Amirhashchi [41]).

Motivated by the above discussions, in this paper, we have investigated Bianchi type-III string cosmological model with bulk viscosity and time-varying cosmological constant. The outline of the paper is as follows. The metric and the field equations are presented in Section 2. Section 3 deals with the solutions of the field equations. In Subsection 3.1 we describe some physical and geometrical properties of the model in the presence of bulk viscosity. In Subsection 3.2 we describe some physical and geometrical features of the model in the absence of bulk viscosity. Finally, in Section 4, concluding remarks are given.

2.The Metric and Field Equations

The line element for general Bianchi type-III space time is considered as

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{-2\alpha x}dy^2 + C^2(t)dz^2, \tag{1}$$

where α is constant. The energy momentum tensor for a cloud of string dust with a bulk viscous fluid of string is given by Letelier [8] and Landau and Lifshitz [42]

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \xi v_{;l}^l \left(g_i^j + v_i v^j\right), \tag{2}$$

where v_i and x_i satisfy the condition

$$v^i v_i = -x^i x_i = -1, \quad v^i x_i = 0, \tag{3}$$

ρ is the proper energy density for a cloud string with particles attached to them, λ is the string tension density of the cloud of strings, $v_{;l}^l = \theta$ is the scalar of expansion, v^i is the four velocity of the particles, and x^i is a unit space-like vector representing the direction of string. If the particle density of the configuration is denoted by ρ_p, then we have-

$$\rho = \rho_p + \lambda \tag{4}$$

The Einstein's field equations (in gravitational units c = 1, G=1) are

$$R_i^j - \frac{1}{2} R\, g_i^j = -8\pi\, T_i^{\,j} + \Lambda g_i^j \,, \qquad (5)$$

where R_i^j is the Ricci tensor; $R = g^{ij}\, R_{ij}$ is the Ricci scalar. In a co-moving co-ordinate system, we have

$$v^i = (0,0,0,1), \qquad x^i = \left(0,0,\frac{1}{C},0\right), \qquad (6)$$

For the metric (1) and energy momentum tensor (2) in the co-moving system of co-ordinates, the field equations (5) yields

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = 8\pi\xi\theta + \Lambda \,, \qquad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = 8\pi\xi\theta + \Lambda \,,$$

(8)

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = 8\pi(\lambda + \xi\theta) + \Lambda \,, \qquad (9)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{\alpha^2}{A^2} = 8\pi\rho + \Lambda \,, \qquad (10)$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0, \qquad (11)$$

where the dot denotes differentiation with respect to time t.

The spatial volume (S^3), the scalar expansion (θ), components of shear tensor (σ_{ij}) and the average anisotropy parameter A_m for the model (1) are given by

$$S^3 = ABC \,, \qquad (12)$$

$$\theta = v_{;l}^l = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \,, \qquad (13)$$

$$\sigma_1^1 = \frac{1}{3}\left(2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right), \qquad (14)$$

$$\sigma_2^2 = \frac{1}{3}\left(2\frac{\dot{B}}{B} - \frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right), \qquad (15)$$

$$\sigma_3^3 = \frac{1}{3}\left(2\frac{\dot{C}}{C} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right), \tag{16}$$

$$\sigma_4^4 = 0. \tag{17}$$

Therefore $$\sigma^2 = \frac{1}{3}\left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC}\right]. \tag{18}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{19}$$

where $\Delta H_i = H_i - H \ (i = 1, 2, 3)$.

3. Solutions of the Field Equations

The field equations (7) – (11) are a system of five equations with seven unknown parameters $A, B, C, \xi, \lambda, \rho$ and Λ. Two additional constraints relating these parameters are required to obtain explicit solutions of the system. We first assume that the expansion (θ) in the model is proportional to the eigen value σ_2^2 of the shear tensor σ_i^j. This condition leads to the following relation between the metric potentials :

$$B = l_1 (AC)^{m_1}, \tag{20}$$

where l_1 and m_1 are arbitrary constants.

Equation (11) leads to

$$A = mB, \tag{21}$$

where m is a positive constant of integration.

From (20) and (21), we obtain

$$B = lC^n, \tag{22}$$

where $l = l_1^{\frac{1}{1-m_1}} m^n, \quad n = \frac{m_1}{1-m_1}$.

Secondly, following Tiwari et al. [40] and Jain et al. [43], we assume that Λ is inversely proportional to S^3

i.e. $$\Lambda = \frac{\beta}{S^3} = \frac{\beta}{ABC}, \tag{23}$$

where β is a positive proportionality constant.

To obtain the determinate model of the universe, we assume that the coefficient of bulk viscosity (ξ) is inversely proportional to expansion scalar (θ). This condition leads to

$$\xi\theta = k, \tag{24}$$

where k is a proportionality constant.

With the help of equations (21) – (24), equation (7) reduces to

$$2\ddot{C} + \left(\frac{2n^2}{n+1}\right)\frac{\dot{C}^2}{C} = \left(\frac{16\pi k}{n+1}\right)C + \frac{2\beta}{ml^2(n+1)}.\frac{1}{C^{2n}}. \tag{25}$$

Let $\dot{C} = f(C)$ which implies that $\ddot{C} = ff'$, where $f' = \frac{df}{dC}$.

Hence equation (25) leads to

$$\frac{d}{dC}\left(f^2\right) + \left(\frac{2n^2}{n+1}\right)\frac{f^2}{C} = \left(\frac{16\pi k}{n+1}\right)C + \frac{2\beta}{ml^2(n+1)}.\frac{1}{C^{2n}}. \tag{26}$$

Equation (26), after integration, reduces to

$$f^2 = \left(\frac{dC}{dt}\right)^2 = \left(\frac{8\pi k}{n^2+n+1}\right)C^2 + \frac{2\beta}{ml^2(1-n)}.\frac{1}{C^{\frac{(2n^2+n-1)}{n+1}}} + \frac{L}{C^{\frac{2n^2}{n+1}}}, \quad n \neq 1, \tag{27}$$

where L is the constant of integration.

With the help of equations (21), (22) and (27), the line element (1) reduces to

$$ds^2 = -\left[\left(\frac{8\pi k}{n^2+n+1}\right)C^2 + \frac{2\beta}{ml^2(1-n)}.\frac{1}{C^{\frac{(2n^2+n-1)}{n+1}}} + \frac{L}{C^{\frac{2n^2}{n+1}}}\right]^{-1} dC^2$$

$$+ m^2 l^2 C^{2n} dx^2 + l^2 C^{2n} e^{-2\alpha x} dy^2 + C^2 dz^2 \tag{28}$$

After using a suitable transformation of coordinates, the line element (28) reduces to

$$ds^2 = -\left[\left(\frac{8\pi k}{n^2+n+1}\right)T^2 + \frac{2\beta}{ml^2(1-n)}.\frac{1}{T^{\frac{(2n^2+n-1)}{n+1}}} + \frac{L}{T^{\frac{2n^2}{n+1}}}\right]^{-1} dT^2$$

$$+ m^2 l^2 T^{2n} dx^2 + l^2 T^{2n} e^{-2\alpha x} dy^2 + T^2 dz^2 \tag{29}$$

3.1 Some Physical and Geometrical Characteristics

The expressions for the energy density ρ, the string tension density λ, the particle density ρ_p, the cosmological constant Λ, the expansion scalar (θ), shear scalar (σ) and spatial volume (V) are, respectively, given by

$$8\pi\rho = \frac{8\pi(n^2+2n)k}{(n^2+n+1)} + \frac{(2n^2+5n-1)\beta}{ml^2(1-n)T^{2n+1}} + \frac{(n^2+2n)L}{T^{\frac{2(n^2+n+1)}{n+1}}} - \frac{\alpha^2}{m^2l^2T^{2n}}, \tag{30}$$

$$8\pi\lambda = \frac{8\pi(2n^2-n-1)k}{(n^2+n+1)} - \frac{(2n+1)\beta}{ml^2T^{2n+1}} + \frac{(n^3+n^2-2n)L}{(n+1)T^{\frac{2(n^2+n+1)}{n+1}}} - \frac{\alpha^2}{m^2l^2T^{2n}}, \tag{31}$$

$$8\pi\rho_p = -\frac{8\pi(n^2-3n-1)k}{(n^2+n+1)} + \frac{6\beta n}{(1-n)ml^2T^{2n+1}} + \frac{(2n^2+4n)L}{(n+1)T^{\frac{2(n^2+n+1)}{n+1}}}, \tag{32}$$

$$\Lambda = \frac{\beta}{ml^2T^{2n+1}}, \tag{33}$$

$$\theta = (2n+1)\left[\frac{8\pi k}{(n^2+n+1)} + \frac{2\beta}{ml^2(1-n)T^{2n+1}} + \frac{L}{T^{\frac{2(n^2+n+1)}{n+1}}}\right]^{\frac{1}{2}}, \tag{34}$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\left[\frac{8\pi k}{(n^2+n+1)} + \frac{2\beta}{ml^2(1-n)T^{2n+1}} + \frac{L}{T^{\frac{2(n^2+n+1)}{n+1}}}\right]^{\frac{1}{2}}, \tag{35}$$

$$V = S^3 = ml^2\, T^{2n+1}. \tag{36}$$

The energy conditions $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied in the presence of bulk viscosity for the metric (29).

From (30), the condition $\rho \geq 0$ leads to

$$(n^2+2n)\left[\frac{8\pi k}{(n^2+n+1)} + \frac{L}{T^{\frac{2(n^2+n+1)}{n+1}}}\right] + \frac{(2n^2+5n-1)\beta}{ml^2(1-n)T^{2n+1}} \geq \frac{\alpha^2}{m^2l^2T^{2n}}, \tag{37}$$

From (32), the condition $\rho_p \geq 0$ leads to

$$\frac{(2n^2+4n)L}{(n+1)T^{\frac{2(n^2+n+1)}{n+1}}}+\frac{6n\beta}{ml^2(1-n)T^{2n+1}}\geq\frac{8\pi(n^2-3n-1)k}{(n^2+n+1)} \tag{38}$$

Form (31), we observe that the string tension density $\lambda \geq 0$ provided

$$\frac{8\pi(2n^2-n-1)k}{(n^2+n+1)}+\frac{(n^3+n^2-2n)L}{(n+1)T^{\frac{2(n^2+n+1)}{n+1}}}\geq\frac{(2n+1)\beta}{ml^2T^{2n+1}}+\frac{\alpha^2}{m^2l^2T^{2n}} \tag{39}$$

It is worth mentioned here that for n > 1, the model does not exist physically realistic as it provides negative energy density from early stage. Therefore, we have concentrated our analysis by considering 0 < n < 1. The spatial volume V is zero at T = 0 and the expansion scalar θ is infinite, which shows that the universe starts evolving with zero volume at T = 0 and hence the space time exhibits point type [44] singularity at the initial epoch. From equations (30) and (34), it is noted that the proper energy density ρ and scalar of expansion θ are decreasing functions of time and they approaches a small positive value at present epoch but never become zero due to the presence of bulk viscosity. This implies that our universe can not be an empty universe even after infinitely large time. From equation (31) it is observed that λ is a deceasing function of time and it is always negative as condition (39) for λ ≥ 0 is not satisfied for $\frac{1}{6}<n<1$. It is pointed out by Letelier [9] that λ may be positive or negative. From equation (33), we observe that Λ is a decreasing function of time and is always positive. Recent cosmological observations [45-52] suggest the existence of a positive cosmological constant Λ with the magnitude Λ $(Gh/c^3) \approx 10^{-123}$. These observations on magnitude and red shift of type Ia supernovae suggest that our universe may be accelerating one with induced cosmological density through the cosmological Λ-term. Thus, our model is consistent with the results of recent observations. We can see from the above discussion that the bulk viscosity plays a significant role in the evolution of the universe. Furthermore, since $\lim_{T\to\infty}\frac{\sigma}{\theta}=\frac{(n-1)}{\sqrt{3}(2n+1)}\neq 0$ as 0 < n < 1, the model never approaches to isotropy, so the Bianchi type-III cosmological model is anisotropic.

3.2 Solution in the Absence of Bulk Viscosity

In the absence of bulk viscosity, i.e. when k = 0, the metric (29) reduces to

$$ds^2 = -\left[\frac{2\beta}{ml^2(1-n)}\cdot\frac{1}{T^{\frac{(2n^2+n-1)}{n+1}}} + \frac{L}{T^{\frac{2n^2}{n+1}}}\right]^{-1} dT^2 + m^2l^2T^{2n}dx^2 + l^2T^{2n}e^{-2\alpha x}dy^2 + T^2dz^2 \quad (40)$$

The physical and kinematical quantities for the model (40) are given by

$$8\pi\rho = \frac{(2n^2+5n-1)\beta}{ml^2(1-n)T^{2n+1}} + \frac{(n^2+2n)L}{T^{\frac{2(n^2+n+1)}{n+1}}} - \frac{\alpha^2}{m^2l^2T^{2n}}, \quad (41)$$

$$8\pi\lambda = -\frac{(2n+1)\beta}{ml^2T^{2n+1}} + \frac{(n^3+n^2-2n)L}{(n+1)T^{\frac{2(n^2+n+1)}{n+1}}} - \frac{\alpha^2}{m^2l^2T^{2n}}, \quad (42)$$

$$8\pi\rho_p = \frac{6n\beta}{ml^2(1-n)T^{2n+1}} + \frac{(2n^2+4n)L}{(n+1)T^{\frac{2(n^2+n+1)}{n+1}}}, \quad (43)$$

$$\Lambda = \frac{\beta}{ml^2T^{2n+1}}, \quad (44)$$

$$\theta = (2n+1)\left[\frac{2\beta}{ml^2(1-n)T^{2n+1}} + \frac{L}{T^{\frac{2(n^2+n+1)}{n+1}}}\right]^{\frac{1}{2}}, \quad (45)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\left[\frac{2\beta}{ml^2(1-n)T^{2n+1}} + \frac{L}{T^{\frac{2(n^2+n+1)}{n+1}}}\right]^{\frac{1}{2}}, \quad (46)$$

$$V = S^3 = ml^2\,T^{2n+1}. \quad (47)$$

In the absence of bulk viscosity, the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied for the model (40). The condition $\rho \geq 0$ leads to

$$\left[\frac{(2n^2+5n-1)\beta}{ml^2(1-n)T} + \frac{(n^2+2n)L}{T^{\frac{2}{n+1}}}\right] \geq \frac{\alpha^2}{m^2l^2}. \quad (48)$$

The string tension density $\lambda \geq 0$ if

$$\left[\frac{(n^3+n^2-2n)L}{(n+1)T^{\frac{2}{n+1}}}-\frac{(2n+1)\beta}{ml^2T}\right]\geq\frac{\alpha^2}{m^2l^2}. \tag{49}$$

We observe that the spatial volume V is zero at T = 0 and the expansion scalar θ is infinite, which shows that the universe starts evolving with zero volume at T = 0 which is big bang scenario. For this model, the scale factors are zero at T = 0, which shows that the space-time exhibits point type [44] singularity. All the physical quantities, proper energy density (ρ), string tension density (λ), particle density ρ_p, shear scalar (σ), expansion scalar (θ) and cosmological constant (Λ) diverge at T = 0. As T → ∞, volume and all the scale factors become infinite where as ρ, λ, ρ_p, Λ, θ, σ tend to zero. Therefore the model would essentially give an empty universe for large time T. Since $\frac{\sigma}{\theta}$ = constant, the model does not approach isotropy for large values of T. Therefore, the model describes a continuously expanding, shearing, non-rotating universe with the big-bang start.

4. Concluding Remarks

In this paper we have presented a new exact solution of Einstein's field equations for anisotropic Bianchi type-III space-time in the presence of bulk viscosity with time varying cosmological constant Λ which is different from the other author's solutions. In general the model is expanding, shearing and non-rotating. The model starts with a big-bang at T=0 and it goes on expanding until it comes out to rest at T= ∞. The initial singularity in the model is point type [44]. Our universe starts evolving with zero volume at T=0 and expand with cosmic time T. In our derived model, we have observed that the cosmological term-Λ decreases as the time increases and it approaches to a small positive value at the present epoch. We observe that $\lim_{T\to\infty}\frac{\sigma}{\theta}\neq 0$, the models do not approach isotropy at any time. It is found that in the presence of bulk viscosity our model never approach empty universe whereas in the absence of bulk viscosity it would essentially give an empty universe for large value of T. We have also discussed some physical and Kinematical properties of the cosmological models.

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